Common Core Standards		**CodeBot Missions**												
Section 1: Mathematics Middle School														
Grade 6		1	2	3	RM	4	5	RM	6	7	RM	8	9	FP
Ratios and Proportional Relationships														
• Understand ratio concepts and use ratio reasoning to solve problems. **Students:**	1. Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."													
	2. Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship. For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."													
	3. Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tapediagrams, double number line diagrams, or equations. a. Make tables of equivalent ratios relating quantities with wholenumber measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. b. Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed? c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent. d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.													
The Number System														
• Apply and extend previous understandings of multiplication and division to divide fractions by fractions. **Students:**	1. Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?													
• Compute fluently with multi-digit numbers and find common factors and multiples. **Students:**	2. Fluently divide multi-digit numbers using the standard algorithm.											X	X	
	3. Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.											X	X	
	4. Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. For example, express 36 + 8 as 4 (9 + 2).													

• Apply and extend previous understandings of numbers to the system of rational numbers. **Students:**	5. Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.					X	X		X			X	X	
	6. Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., –(–3) = 3, and that 0 is its own opposite. b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.													
	7. Understand ordering and absolute value of rational numbers. a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. For example, interpret –3 > –7 as a statement that –3 is located to the right of –7 on a number line oriented from left to right. b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. For example, write –3 oC > –7 oC to express the fact that –3 oC is warmer than –7 oC. c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. For example, for an account balance of –30 dollars, write |–30| = 30 to describe the size of the debt in dollars. d. Distinguish comparisons of absolute value from statements about order. For example, recognize that an account balance less than –30 dollars represents a debt greater than 30 dollars.								X	X			X	
	8. Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.													
Expressions and Equations														
• Apply and extend previous understandings of arithmetic to algebraic expressions.	1. Write and evaluate numerical expressions involving whole-number exponents.													

Students:	2. Write, read, and evaluate expressions in which letters stand for numbers. a. Write expressions that record operations with numbers and with letters standing for numbers. For example, express the calculation "Subtract y from 5" as 5 – y. Common Core State Standards for MATHEMATICS grade 6 | 44 b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms. c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving wholenumber exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). For example, use the formulas V = s3 and A = 6 s2 to find the volume and surface area of a cube with sides of length s = 1/2.					X	X		X	X		X	X	
	3. Apply the properties of operations to generate equivalent expressions. For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.													
	4. Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.													
• Reason about and solve one-variable equations and inequalities. **Students:**	5. Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.					X	X		X	X		X	X	
	6. Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.					X	X		X	X		X	X	
	7. Solve real-world and mathematical problems by writing and solving equations of the form x + p = q and px = q for cases in which p, q and x are all nonnegative rational numbers.													
	8. Write an inequality of the form x > c or x < c to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form x > c or x < c have infinitely many solutions; represent solutions of such inequalities on number line diagrams.					X	X		X	X		X	X	
• Represent and analyze quantitative relationships between dependent and independent variables. **Students:**	9. Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.					X	X		X	X		X	X	
Geometry														

• Solve real-world and mathematical problems involving area, surface area, and volume. **Students:**	1. Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.													
	2. Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas V = l w h and V = b h to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.													
	3. Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.													
	4. Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.													
Statistics and Probability														
• Develop understanding of statistical variability. **Students:**	1. Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. For example, “How old am I?” is not a statistical question, but “How old are the students in my school?” is a statistical question because one anticipates variability in students’ ages.													
	2. Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.													
	3. Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.													
• Summarize and describe distributions. **Students:**	4. Display numerical data in plots on a number line, including dot plots, histograms, and box plots.													
	5. Summarize numerical data sets in relation to their context, such as by: a. Reporting the number of observations. b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement. c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.													
Grade 7		1	2	3	RM	4	5	RM	6	7	RM	8	9	FP
Ratios and Proportional Relationships														
• Analyze proportional relationships and use them to solve real-world and mathematical problems. **Students:**	1. Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction 1/2/1/4 miles per hour, equivalently 2 miles per hour.													

	2. Recognize and represent proportional relationships between quantities. a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. c. Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn. d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.								X			X	X	
	3. Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.											X	X	
The Number System														
• Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers. **Students:**	1. Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. a. Describe situations in which opposite quantities combine to make 0. For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged. b. Understand p + q as the number located a distance |q| from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. c. Understand subtraction of rational numbers as adding the additive inverse, p – q = p + (–q). Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. d. Apply properties of operations as strategies to add and subtract rational numbers.					X			X			X	X	
	2. Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then –(p/q) = (–p)/q = p/(–q). Interpret quotients of rational numbers by describing realworld contexts. c. Apply properties of operations as strategies to multiply and divide rational numbers. d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.					X			X	X		X	X	
	3. Solve real-world and mathematical problems involving the four operations with rational numbers.					X			X	X		X	X	
Expressions and Equations														

• Use properties of operations to generate equivalent expressions. **Students:**	1. Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.													
	2. Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05."													
• Solve real-life and mathematical problems using numerical and algebraic expressions and equations. **Students:**	3. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.					X	X		X	X		X	X	
	4. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. a. Solve word problems leading to equations of the form px + q = r and p(x + q) = r, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width? b. Solve word problems leading to inequalities of the form px + q > r or px + q < r, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.					X	X		X	X		X	X	
Geometry														
• Draw, construct and describe geometrical figures and describe the relationships between them. **Students:**	1. Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.													
	2. Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.													
	3. Describe the two-dimensional figures that result from slicing threedimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.													
• Solve real-life and mathematical problems involving angle measure, area, surface area, and volume. **Students:**	4. Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.										X			
	5. Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.													
	6. Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.													
Statistics and Probability														

• Use random sampling to draw inferences about a population. **Students:**	1. Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.													
	2. Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.													
• Draw informal comparative inferences about two populations. **Students:**	3. Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.													
	4. Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.													
• Investigate chance processes and develop, use, and evaluate probability models. **Students:**	5. Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.													
	6. Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.													
	7. Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected. b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?													

	8. Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event. c. Design and use a simulation to generate frequencies for compound events. For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?													
Grade 8		1	2	3	RM	4	5	RM	6	7	RM	8	9	FP
The Number System														
• Know that there are numbers that are not rational, and approximate them by rational numbers. **Students:**	1. Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.													
	2. Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π2). For example, by truncating the decimal expansion of √2, show that √2 is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.													
Expressions and Equations														
• Work with radicals and integer exponents. **Students:**	1. Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, 32 × 3–5 = 3–3 = 1/33 = 1/27.													
	2. Use square root and cube root symbols to represent solutions to equations of the form x2 = p and x3 = p, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that √2 is irrational.													
	3. Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. For example, estimate the population of the United States as 3 × 108 and the population of the world as 7 × 109, and determine that the world population is more than 20 times larger.													
	4. Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.													
• Understand the connections between proportional relationships, lines, and linear equations. **Students:**	5. Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.													

	6. Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation y = mx for a line through the origin and the equation y = mx + b for a line intercepting the vertical axis at b.													
• Analyze and solve linear equations and pairs of simultaneous linear equations. **Students:**	7. Solve linear equations in one variable. a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form x = a, a = a, or a = b results (where a and b are different numbers). b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.													
	8. Analyze and solve pairs of simultaneous linear equations. a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. For example, 3x + 2y = 5 and 3x + 2y = 6 have no solution because 3x + 2y cannot simultaneously be 5 and 6. c. Solve real-world and mathematical problems leading to two linear equations in two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.													
Functions														
• Define, evaluate, and compare functions. **Students:**	1. Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.1													
	2. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.													
	3. Interpret the equation y = mx + b as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. For example, the function A = s2 giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.													
• Use functions to model relationships between quantities. **Students:**	4. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.													
	5. Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.													
Geometry														

• Understand congruence and similarity using physical models, transparencies, or geometry software. **Students:**	1. Verify experimentally the properties of rotations, reflections, and translations: a. Lines are taken to lines, and line segments to line segments of the same length. b. Angles are taken to angles of the same measure. c. Parallel lines are taken to parallel lines.													
	2. Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.													
	3. Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.													
	4. Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar twodimensional figures, describe a sequence that exhibits the similarity between them.													
	5. Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.													
• Understand and apply the Pythagorean Theorem. **Students:**	6. Explain a proof of the Pythagorean Theorem and its converse.													
	7. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.													
	8. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.													
• Solve real-world and mathematical problems involving volume of cylinders, cones and spheres.	9. Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.													
Statistics and Probability														
• Investigate patterns of association in bivariate data. **Students:**	1. Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.													
	2. Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.													
	3. Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.													
	4. Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?													

Section 3: Mathematics High School														
Algebra		1	2	3	RM	4	5	RM	6	7	RM	8	9	FP
Seeing Structure in Expressions														
• Interpret the structure of expressions **Students:**	1. Interpret expressions that represent a quantity in terms of its context.★ a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret P(1+r)n as the product of P and a factor not depending on P.					X	X		X	X		X	X	
	2. Use the structure of an expression to identify ways to rewrite it. For example, see x4 – y4 as (x2)2 – (y2)2, thus recognizing it as a difference of squares that can be factored as (x2 – y2)(x2 + y2).													
• Write expressions in equivalent forms to solve problems **Students:**	3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★ a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. c. Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15t can be rewritten as (1.151/12)12t ≈ 1.01212t to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.													
	4. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments.★													
Arithmetic with Polynomials and Rational Expressions														
• Perform arithmetic operations on polynomials **Students:**	1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.													
• Understand the relationship between zeros and factors of polynomials **Students:**	2. Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x).													
	3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.													
• Use polynomial identities to solve problems Students:	4. Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity (x2 + y2)2 = (x2 – y2)2 + (2xy)2 can be used to generate Pythagorean triples.													
	5. (+) Know and apply the Binomial Theorem for the expansion of (x + y)n in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.1													
• Rewrite rational expressions **Students:**	6. Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.													
	7. (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.													
Creating Equations														

• Create equations that describe numbers or relationships **Students:**	1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.					X	X		X	X		X	X	
	2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.													
	3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.													
	4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law V = IR to highlight resistance R.													
Reasoning with Equations and Inequalities														
• Understand solving equations as a process of reasoning and explain the reasoning **Students:**	1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.					X	X		X	X		X	X	
	2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.													
• Solve equations and inequalities in one variable **Students:**	3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.					X	X		X	X		X	X	
	4. Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form (x – p)2 = q that has the same solutions. Derive the quadratic formula from this form. b. Solve quadratic equations by inspection (e.g., for x2 = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b.													
• Solve systems of equations **Students:**	5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.													
	6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.													
	7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line y = –3x and the circle x2 + y2 = 3													
	8. (+) Represent a system of linear equations as a single matrix equation in a vector variable.													
	9. (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).													
• Represent and solve equations and inequalities graphically	10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).													

Students:	11. Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★													
	12. Graph the solutions to a linear inequality in two variables as a halfplane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.													

Geometry		1	2	3	RM	4	5	RM	6	7	RM	8	9	FP
Congruence														
• Experiment with transformations in the plane. **Students:**	1. Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.													
	2. Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).													
	3. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.													
	4. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.													
	5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.													
• Understand congruence in terms of rigid motions. **Students:**	6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.													
	7. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.													
	8. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.													
• Prove geometric theorems. **Students:**	9. Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints													
	10. Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.													
	11. Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals													

• Make geometric constructions. **Students:**	12. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.													
	13. Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.													
Similarity, Right Triangles, and Trigonometry														
• Understand similarity in terms of similarity transformations **Students:**	1. Verify experimentally the properties of dilations given by a center and a scale factor: a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.													
	2. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.													
	3. Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.													
• Prove theorems involving similarity **Students:**	4. Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.													
	5. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.													
• Define trigonometric ratios and solve problems involving right triangles **Students:**	6. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.													
	7. Explain and use the relationship between the sine and cosine of complementary angles.													
	8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.★													
• Apply trigonometry to general triangles **Students:**	9. (+) Derive the formula A = 1/2 ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.													
	10. (+) Prove the Laws of Sines and Cosines and use them to solve problems.													
	11. (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).													
Circles														
• Understand and apply theorems about circles **Students:**	1. Prove that all circles are similar.													
	2. Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.													
	3. Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.													
	4. (+) Construct a tangent line from a point outside a given circle to the circle.													

• Find arc lengths and areas of sectors of circles **Students:**	5. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector													
Expressing Geometric Properties with Equations														
• Translate between the geometric description and the equation for a conic section **Students:**	1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.													
	2. Derive the equation of a parabola given a focus and directrix.													
	3. (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.													
• Use coordinates to prove simple geometric theorems algebraically **Students:**	4. Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).													
	5. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).													
	6. Find the point on a directed line segment between two given points that partitions the segment in a given ratio.													
	7. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.★													
Geometric Measurement and Dimension														
• Explain volume formulas and use them to solve problems. **Students:**	1. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.											X		
	2. (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.													
	3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.★													
• Visualize relationships between twodimensional and three-dimensional objects **Students:**	4. Identify the shapes of two-dimensional cross-sections of threedimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.													
Modeling with Geometry														
• Apply geometric concepts in modeling situations. **Students:**	1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).★													
	2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).★													
	3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).★											X	X	